

1 In this question take $g = 10$.

A golf ball is hit from ground level over horizontal ground. The initial velocity of the ball is 40 m s^{-1} at an angle α to the horizontal, where $\sin \alpha = 0.6$ and $\cos \alpha = 0.8$. Air resistance may be neglected.

(i) Find an expression for the height of the ball above the ground t seconds after projection. [2]

(ii) Calculate the horizontal range of the ball. [4]

2 A ball is kicked from ground level over horizontal ground. It leaves the ground at a speed of 25 m s^{-1} and at an angle θ to the horizontal such that $\cos \theta = 0.96$ and $\sin \theta = 0.28$.

(i) Show that the height, y m, of the ball above the ground t seconds after projection is given by $y = 7t - 4.9t^2$. Show also that the horizontal distance, x m, travelled by this time is given by $x = 24t$. [3]

(ii) Calculate the maximum height reached by the ball. [2]

(iii) Calculate the times at which the ball is at half its maximum height.

Find the horizontal distance travelled by the ball between these times. [4]

(iv) Determine the following when $t = 1.25$.

(A) The vertical component of the velocity of the ball.

(B) Whether the ball is rising or falling. (You should give a reason for your answer.)

(C) The speed of the ball. [5]

(v) Show that the equation of the trajectory of the ball is

$$y = \frac{0.7x}{576} (240 - 7x).$$

Hence, or otherwise, find the range of the ball. [5]

- 3 A particle is thrown vertically upwards and returns to its point of projection after 6 seconds. Air resistance is negligible.

Calculate the speed of projection of the particle and also the maximum height it reaches. [4]

- 4 You should neglect air resistance in this question.

A small stone is projected from ground level. The maximum height of the stone above horizontal ground is 22.5 m.

- (i) Show that the vertical component of the initial velocity of the stone is 21 m s^{-1} . [2]

The speed of projection is 28 m s^{-1} .

- (ii) Find the angle of projection of the stone. [2]
- (iii) Find the horizontal range of the stone. [4]

5 In this question take the value of g to be 10 m s^{-2} .

A particle A is projected over horizontal ground from a point P which is 9 m above a point O on the ground. The initial velocity has horizontal and vertical components of 10 m s^{-1} and 12 m s^{-1} respectively, as shown in Fig. 7. The trajectory of the particle meets the ground at X. Air resistance may be neglected.

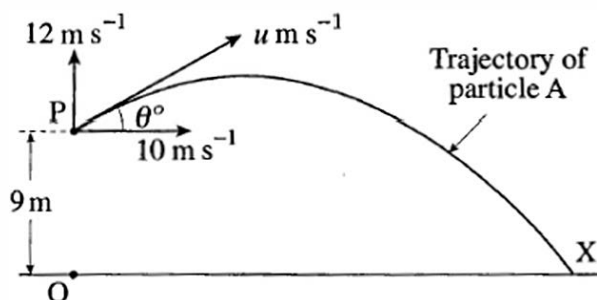


Fig. 7

- (i) Calculate the speed of projection $u \text{ m s}^{-1}$ and the angle of projection θ° . [3]
- (ii) Show that, t seconds after projection, the height of particle A above the ground is $9 + 12t - 5t^2$. Write down an expression in terms of t for the horizontal distance of the particle from O at this time. [4]
- (iii) Calculate the maximum height of particle A above the point of projection. [2]
- (iv) Calculate the distance OX. [4]

A second particle, B, is projected from O with speed 20 m s^{-1} at 60° to the horizontal. The trajectories of A and B are in the same vertical plane. Particles A and B are projected at the same time.

- (v) Show that the horizontal displacements of A and B are always equal. [2]
- (vi) Show that, t seconds after projection, the height of particle B above the ground is $10\sqrt{3}t - 5t^2$. [1]
- (vii) Show that the particles collide 1.7 seconds after projection (correct to two significant figures). [3]

6 Ali is throwing flat stones onto water, hoping that they will bounce, as illustrated in Fig. 5.

Ali throws one stone from a height of 1.225 m above the water with initial speed 20 m s^{-1} in a horizontal direction. Air resistance should be neglected.

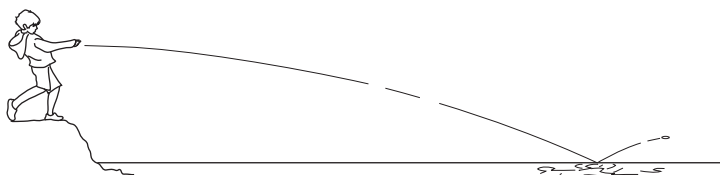


Fig. 5

- (i) Find the time it takes for the stone to reach the water. [2]
- (ii) Find the speed of the stone when it reaches the water and the angle its trajectory makes with the horizontal at this time. [5]

- 7 A projectile P travels in a vertical plane over level ground. Its position vector \mathbf{r} at time t seconds after projection is modelled by

$$\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 30 \\ 40 \end{pmatrix} t - \begin{pmatrix} 0 \\ 5 \end{pmatrix} t^2,$$

where distances are in metres and the origin is a point on the level ground.

(i) Write down

(A) the height from which P is projected,

(B) the value of g in this model.

[2]

(ii) Find the displacement of P from $t = 3$ to $t = 5$.

[2]

(iii) Show that the equation of the trajectory is

$$y = 5 + \frac{4}{3}x - \frac{x^2}{180}.$$

[4]